Algebraic Proofs of Nonlocality

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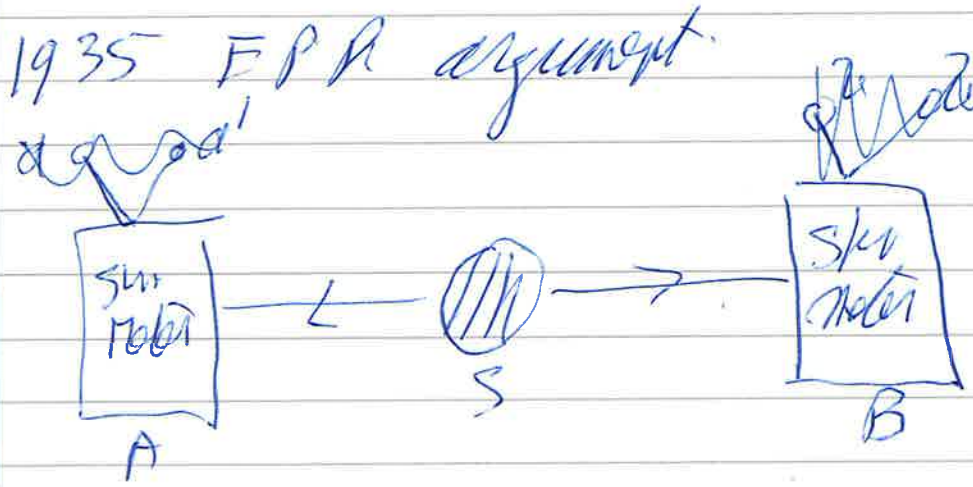
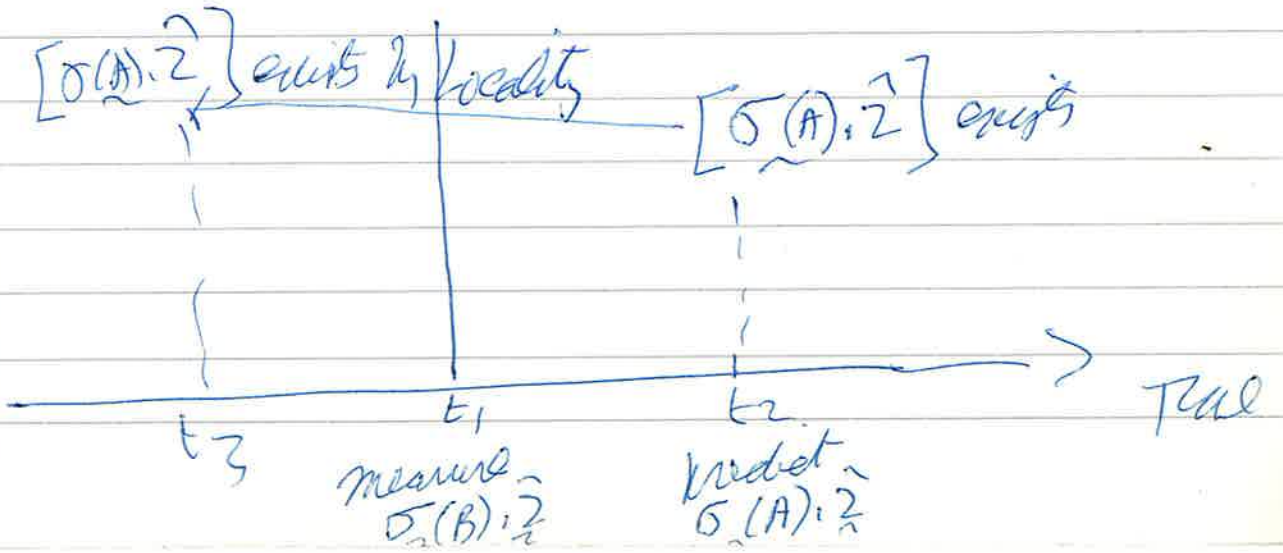
In what sense is QM a nonlocal theory? — cp 2 slit experiment, extended wave-function, Efimov states in nuclear physics, &c.

But RQFT is based on causality axioms, & how can the theory be nonlocal?

(1) 1935 EPR argument.



Set $\hat{a} \parallel \hat{b} \parallel \hat{z}$ $\quad \Psi_{singlet} = \frac{1}{\sqrt{2}} \{ |\sigma(A)\cdot\hat{z} = +1\rangle |\sigma(B)\cdot\hat{z} = -1\rangle - |\sigma(A)\cdot\hat{z} = -1\rangle |\sigma(B)\cdot\hat{z} = +1\rangle \}$



(2)

So Einstein Dilemma

F → ~(L) v Incompleteness

∴ QM formalism ⟹ nonlocality or Incompleteness.

↓

completed version of QM (hidden variables)

↓ Bell Inequalities

↓ violated by Expt.

nonlocality.

So QM is nonlocal simpliciter.

But Bohr denied the nonlocality was a "physical" effect.
Since reject Bohr's approach

then what about
? + Locality + hidden variables ⟹ Bell Ineq.

Early proofs Bell (1964) assume Determinism
+ probability structure
(J.P. for incompatible observables) + Also auxiliary assumptions cf. Lo-Shimony cell

Two Controversies (1) Does proof of Bell, under determinism, commit one to J.D.?
(after Stapp 1971, Eberhard 1977) Fine says yes (1982)
Redhead says no (1983, 1988)
with Svetlichny, Butterfield, Bacon.

Fine's 1982 theorem extended to beautiful mathematical theory by Pitowski and others — see Pitowski (1989) Quantum Probability – Quantum Logic

Generalized Bell inequalities are facet inequalities defining the facets of a multidimensional polytope.

(2) ~~Does~~ Can the Stapp–Eberhard proof be extended to indeterminism.

Stapp says yes

Hellman (1982) Redhead (Clifton, Butterfield) says no.
Redhead (1983, 1987).
CBR paper 1990.
A Stapp in the Wrong Direction

Can we give proofs of nonlocality in h.v. reconstructions that do not use probability theory?

Answer Assuming determinism Yes (nearly)
Assuming indeterminism NO
But there do now exist ~~algebraic~~ so-called stochastic algebraic proofs of nonlocality (Elby, Clifton)
These proceed by starting with general stochastic framework and showing certain probs are 0 or 1 ~~then~~ then using these assignments to get an algebraic contradiction.

History of the algebraic proofs

1.) Project: Derive a Kochen-Specker (1967) contradiction for two spin ½ systems. (cf Gleason (1957))

1º. show local observables like $\sigma(A)\cdot\hat{a}$ must depend on a total context of properties of the whole system.

Proposed by Bub in 1976 in form of a question.
Cited Maczynski's theorem (1971) as showing that it be extendable from maximal to locally maximal observables. ?

But M's theorem was so extended by Demopoulos, Humphreys and Bub in 1980.

So no algebraic proof of nonseparability existed 20 years.

2) In 1983 Redhead's disproof derived a K–S paradox on one of a pair of spin-one particles assuming separability and locality.
Involved locally non-maximal observables.

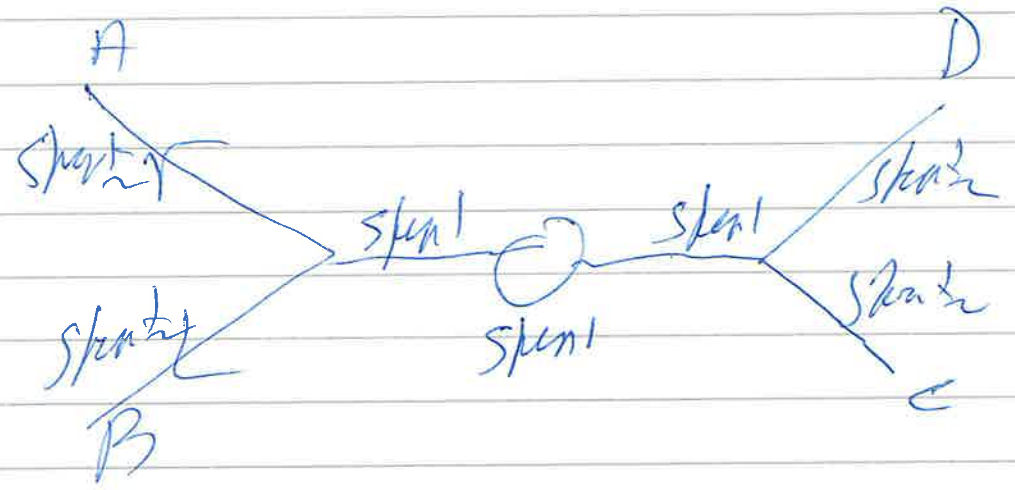
3) Stairs (1983) followed by Brown & Svetlichny (1990)

showed how to get a proof of nonlocality using similar assumptions to RdH, but extended to locally maximal observables.
but again assuming determinism.

4) Eeby produced a stochastic version of Spann – Braun – Svozilhelat

5) Greenberger, Horne & Zeilinger (1989) produced a brand new "algebraic" proof quite unrelated to K–S & Gleason.

6) 1990 Redhead & Clifton showed the stated proof given by Greenberger could not be made to work but Clifton devised a modified proof that does work.



then it is shown that if $\theta_A + \theta_B - \theta_C - \theta_D = 0$,

$$A(\theta_A)\,B(\theta_B)\,C(\theta_C)\,D(\theta_D) = -1$$

and if $\theta_A + \theta_B - \theta_C - \theta_D = \pi$, $A(\theta_A)B(\theta_B)C(\theta_C)D(\theta_D) = +1$

Consider 5 possible settings for $\{\theta_A\ \theta_B\ \theta_C\ \theta_D\}$

	(1)	(2)	(3)	(4)	Also (5)
θ_A	→	↓	→	↑	{↓}
θ_B	→	↑	↑	→	→ / ↑
θ_C	→	→	↑	↑	
θ_D	→	→	→	→	→
	ABCD = −1	ABCD = −1	ABCD = −1	ABCD = −1	ABCD = +1

From (4) & 5)
$A(\uparrow) = -A(\downarrow)$

Hence we obtain

(1) × (2), (1) × (3) similarly, (1), (4) and

$$A(\rightarrow)B(\rightarrow) = A(\downarrow)B(\uparrow)$$
$$B(\rightarrow)C(\rightarrow) = B(\uparrow)C(\uparrow)$$
$$A(\rightarrow)C(\rightarrow) = A(\uparrow)C(\uparrow)$$

Multiplying $1 = A(\downarrow)A(\uparrow)$

or $A(\downarrow) = A(\uparrow)$, But $A(\downarrow) = -A(\uparrow)$ from (4) & (5)

∴ contradiction

Greenberger considers their argument a counterargument to Bohr's reply to EPR. He talks of restricting the discussion to the "super-classical case" "where an element of reality exists by virtue of perfect predictability"

But this is wrong as the quantities

$A(\rightarrow)$, $A(\uparrow)$ $A(\downarrow)$, $B(\rightarrow)$, $B(\uparrow)$, $C(\rightarrow)$, $C(\uparrow)$, $D(\rightarrow)$

involved in the argument do not all exist in the same experimental context, so counterfactual formulations of locality are ignored, and other smuggles in definiteness.

7) Clifton has generalized the GHZ argument to the stochastic case, in a manner parallel to what Elby did for Stairs

So where are we left?

correlations that cannot be explained

not causal

cf Redhead's [illegible] Redhead (1986) and non-signalling theorems of Ghirardi Rimini, Weber (1980) and others (including Eberhard, Page, Shimony, Redhead etc).

Recent work on violations of Bell Inequalities in R&FT derived by eg. Summers, Werner Landau Fredenhagen etc. - [illegible] the proof
See also p.115 of my book.